AN EXPRESS-ESTIMATE OF THE RAIL ACCELERATOR PARAMETERS

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Quite extensive literature of both the survey [1, 2] and specific kind [3, 4] exists apropos the electromagnetic acceleration of bodies. The majority of the results described in these works is obtained by using numerical methods. Their analytic extension to the whole gamut of modifications of such a multiparameter problem as is the problem of the rail accelerator, turns out to be made difficult here. In which connection it is useful to find those relationships connecting the characteristics of the discharge loop with the parameters of the body being accelerated, which would permit making equally likely estimates of quantities of interest without pretensions to special accuracy but qualitatively correctly and rapidly. It should be noted that important relationships concerning taking account of the influence of the magnitude of the loop active resistance on the asymptotic of the efficiency of transforming the storage energy into kinetic energy are presented in [5]. The present paper proposes to make accessible a rapid estimate and magnitudes of the electrical characteristics of the discharge loop as a function of the requirements imposed on the acceleration channel (the length of the acceleration, the allowable overloads, the velocity required) for both a single-stage and multistage accelerator.

1. The Lagrange function $\mathscr{L}$, that has the following form for an ideal rail-accelerator

$$\mathscr{L} = \frac{m\dot{q}_1^2}{2} + \frac{(L_0 + kq_1)}{2}\dot{q}_2^2 - \frac{q_2^2}{2C}, \qquad (1.1)$$

is used for formulation of the problem. Here the generalized coordinates q_1 and q_2 are, respectively, the path traversed by the body and the charge of the energy storage condenser, L_0 is the initial accelerator inductance, C is the condenser capacitance, k is the linear inductance of the accelerator, and m is the mass of the body being accelerated.

The analytic solution of the equations of motion generated by (1.1) has not been found and, as was mentioned above, a theoretical examination of the acceleration process requires the numerical solution of these equations.

The following procedure is proposed below for an express-estimate of the rail accelerator parameters. Let us consider the "motion" equation for q_2

$$\ddot{q}_2 L + \dot{q}_2 \dot{L} + q_2/C = 0 \qquad (1.2)$$

($L = L_0 + kq_1$). In conformity with (1.2) let us formulate the equation

$$\ddot{q} + \frac{2n}{(n+2)t_*}\dot{q} + \frac{2q}{L_0C(n+2)} = 0, \qquad (1.3)$$

in which the time-dependent coefficients of (1.2) are replaced by constants that represent the average over the acceleration time $t_* = x_*/\langle V\rangle$ (x_* is the acceleration path, and $\langle V\rangle$ is the mean velocity on the acceleration section). The averaging is performed from the computation that the increment in the inductance ΔL would be a quantity nL_0 during the time t_*. The functions

$$q = \frac{q_0\omega_0}{\omega}\exp(-\delta t/2)\cos(\omega t - \varphi); \qquad (1.4a)$$

$$\dot{q} = \frac{q_0\omega_0^2}{\omega}\exp(-\delta t/2)\sin\omega t, \qquad (1.4b)$$

$$\delta = 2n/(n+2)t_*, \quad \omega_0 = \omega_{00}\sqrt{2/(n+2)}, \quad \omega_{00} = 1/\sqrt{L_0C},$$

$$\varphi = \operatorname{arctg}(\delta/2\omega), \quad \omega = \sqrt{\omega_0^2 - \delta^2/4}$$

are a solution of (1.3).

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The selection of namely the vibrational solution of (1.3) is dictated by the fact that the presence of a component proportional to the current in (1.3) does not result in irreversible losses (as for a purely active resistance) responsible for the passage over to an aperiodic solution. For an ideal rail accelerator the quasivibrational discharge mode is realized independently of the value of the coefficient in the mentioned component. We shall consider (1.4a) and (1.4b) to represent the approximate solution (1.2). We use the energy conservation law for the subsequent calculations and (in the interests of simplifying the calculations) make them for the times $t_* = \lambda\pi/\omega$. Then we can write for $\lambda = (2p + 1)/2$, $p = 0, 1, 2, \ldots$,

$$L_* \dot{q}_*^2/2 + mV_*^2/2 + q_*^2/2C = E_0; \tag{1.5a}$$

and for $\lambda = 1, 2, 3, \ldots$

$$q_*^2/2C + mV_*^2/2 = E_0 \tag{1.5b}$$

(the quantities with the asterisk refer to the acceleration checking time t_*). Let us supplement these relationships by a kinetic condition implying equal-acceleration of the speed-up process

$$t_* = 2x_*/V_* = 2nL_0/kV_*. \tag{1.6}$$

The following reasoning is a specific justification. The quadratic dependence of the force acting on the body on the magnitude of the current results, for a quasisinusoidal dependence of the current on the time, in that dependence of the velocity of the body being accelerated on the time as could be represented in the form of sections continuing each other and diminishing along the ordinate, and each of them recalls a hysteresis-free magnetization curve. During the time of each such section the influence of the acceleration growing in its first half (on the body velocity) is compensated almost completely by the acceleration drop in the second. Consequently, the introduction of a mean effective acceleration over time is a fair approximation to obtain a number of integral relationships. The constraints associated with such an approximation are discussed below. Numerical computations performed confirmed this and showed that the accuracy of the relationship (1.6) turns out to be satisfactory to $n = 2, \ldots, 4$.

After the calculations we obtain expressions for the process efficiency corresponding to (1.5a) and (1.5b)

$$\eta \equiv \frac{mV_*^2}{2E_0} = 1 - 2\frac{n+1}{n+2}\frac{(4\lambda^2\pi^2 + \beta^2)\,e^{-\beta}}{4\lambda^2\pi^2} - \frac{\beta^2 e^{-\beta}}{4\pi^2\lambda^2}; \tag{1.7}$$

$$\eta = 1 - e^{-\beta}, \tag{1.8}$$

as well as the connection between the natural frequency of the storage loop and the acceleration characteristics

$$\omega_{00} = \frac{V_*}{x_* 4\sqrt{2}}\sqrt{(4\lambda^2\pi^2 + \beta^2)(n+2)} \tag{1.9}$$

($\beta = 2n/(n + 2)$). Computations showed that the acceleration efficiency drops noticeably for $n > 2$ and a further increase in the velocity requires an out-of-proportion magnification of the increase in the acceleration length. Substituting $n = 2$ into (1.7)-(1.9) and noting that for $\lambda = 1$, $4\pi^2\lambda^2 \gg \beta^2$, we already obtain $\eta = 1 - 1.5\exp(-1)$ for $\lambda = (2p + 1)/2$, $p = 0, 1, 2, \ldots$, for $\eta = 1 - \exp(-1)$ for $\lambda = 1, 2, \ldots$ and

$$\omega_{00} = \lambda\pi V_*/(\sqrt{2}\, x_*). \tag{1.10}$$

The last two formulas for the efficiency can be combined into one by taking their average and being released in such a way from the relation to λ. Then for $n = 2$

$$mV_*^2/2E_0 = 0.5. \tag{1.11}$$

It was useful to compare the results of the estimates obtained by means of (1.10) and (1.11) with the results of the numerical solution of the system of differential equations generated by

TABLE 1

$\varkappa$	η	λ_{est}	λ_{true}	$\varkappa$	η	λ_{est}	λ_{true}
0,15	0,4	2,6	3	2,4	0,55	0,64	0,8
0,24	0,48	2	2,6	5,6	0,59	0,4	0,5
0,4	0,45	1,6	1,9	56	0,31	0,5	0,5
0.56	0.46	1.3	1.6				

(1.1). To this end, an appropriate program was compiled and several modifications were computed. We use the quantity $\varkappa = (CU_0k)^2/mL_0$ (U_0 is the initial voltage on the condenser). It is easy to show (by analyzing the conditions for satisfying the inequality $\varphi \ll 1$) that for $\varkappa \ll 1$ the body being accelerated is "heavy" for this accelerator, its acceleration is realized slowly for a quasiperiodic discharge of the condenser during several periods. And on the other hand, for $\varkappa \gg 1$ the body is "light" and is accelerated in a time approximately equal to the duration of the first half-period of the discharge, which in this case is almost aperiodic in nature (with, nevertheless, certain current passages through zero). The results of the comparison (Table 1) show that the quantities of the efficiency obtained for n = 2 differ slightly from the value 0.5 given by (1.11) for almost all $\varkappa$. The deviation is noticeable just for $\varkappa > 50$, where neglecting the quantity β as compared with $2\lambda\pi$ becomes incorrect and η, evaluated by means of (1.7) equals 0.35. Let us also note that the data of Table 1 correspond to the results in [1]. To analyze the accuracy of (1.10) we convert it by taking account of (1.11)

$$\lambda = \varkappa^{-1/2}. \tag{1.12}$$

Values of λ_{est} calculated by (1.12) and λ_{true} determined from the results of the numerical solution are presented in Table 1. The number of half-periods together with fractions of the last half-period of the discharge proceeding up to the time the quantity ΔL reaches the value $2L_0$ was taken as λ_{true}. As is seen, the nearness of the estimates to the "exact" numerical results is totally satisfactory. Let us note that the relationship (1.12) permits estimation of the number of roots of the solution $\dot{q}_2(t)$ from the system of equations generated by (1.1) in the interval $0 \le t \le t_*$ corresponding to the change in the coordinate q_1 within the limits from 0 to $2L_0/k$ without solving the system itself.

Therefore, by taking the average of the coefficients of the quasilinear equation (1.2) with (1.6) taken into account, and using the energy conservation law, two relations are successfully obtained that connect the energy storage loop characteristics (ω_{00}, E_0), the parameters (x_*, k) of the acceleration channel, and the kinetic energy of the body being accelerated. Let us recall that these relationships are approximate and have a limit of their application. The presence of such a limit is governed by the fact that the accuracy of the assumption expressed by (1.6) drops as λ increases, an artificial reduction in the velocity V_* occurs. The relationships (1.7) and (1.8) also demonstrate this.

Indeed as $\lambda \to \infty$ and $n \to \infty$ the efficiency calculated by them does not tend to one as it should but is a somewhat smaller value. Nevertheless the mentioned relations, as is seen from Table 1, turn out to be completely suitable for a quite extensive domain of variation of $\varkappa$ and n, which is interesting from the practical viewpoint; they even reflect the non-monotonic nature of the dependence $\eta(\varkappa)$, inherent to the acceleration in an ideal rail accelerator and illustrated in [1].

2. By using the method described above we estimate the possibility (or requirement) of a multistage rail accelerator each of whose stages operates independently of the others. Indeed by supplementing (1.5a) or (1.5b) with the quantities $mV_i^2/2$ characterizing the kinetic energy of the body at the entrance to the (i + 1)-th stage and by writing (1.6) in the form

$$t_* = \frac{2x_*}{V_i + V_{i+1}} = \frac{2nL_0}{k(V_i + V_{i+1})}, \tag{2.1}$$

we obtain from the energy conservation law (again inaccurate for n, $\lambda \to \infty$)

$$\nu_{i+1}^2 - 1 = \frac{2E_0\eta}{mV_i^2}. \tag{2.2}$$

Hence

$$\nu_{i+1} + 1 = \frac{\omega_{00} x_*}{V_i} F \tag{2.3}$$

becomes a corollary of (2.1), where η is determined from (1.7) or (1.8), $F = 2\sqrt{2}/(\lambda\pi\sqrt{n+2})$; $V_{i+1} = \nu_{i+1}V_i$ is the velocity at the exit of the (i + 1)-th accelerator stage. We again have two relationships, one of which is the potential possibility of this stage while the other formulates a requirement on the eigenfrequency of the discharge loop (this means on the initial voltage U_{i+1} on the condenser) as a function of the given acceleration characteristics

$$U_{i+1} = V_i(\nu_{i+1} + 1)\sqrt{2E_0 B} \tag{2.4}$$

($B = x_*^2F^2/L_0$). It is easy to see that (2.3) goes over into (1.9) for $V_0 = 0$ (acceleration with zero initial velocity). The accuracy of (2.2) and (2.3) is also verified during analysis of the results of the numerical solution of the equations of motion generated by (1.1).

The discrepancies between the corresponding quantities in each stage are about 1%. The qualitative deduction characteristic for both computations using (2.2) and (2.3) and for numerical computations, and that merits attention, reduces to the following. If two multistage accelerators with initial voltage U_i in each stage and with velocity V_i achievable in each stage are compared for different velocities at the entrance to the first stage, then it turns out that if the ratio between the initial velocities at the entrance to the first stage is j then starting with the stage numbered i = j the difference between the body velocities at their exit, exactly as the difference in the initial voltages of the batteries of these stages, is around 5%. In other words, the difference between multistage accelerator parameters that owes its origin just to the difference between the velocities at the entrance to the first stage, exists just in the first several stages (see Fig. 1). Superposed in Fig. 1 are discrete dependences of the reduced velocity at the stage exit and the initial voltage of the battery of condensers of the stage on the number of the stage for four modifications that differ by the velocities at the entrance to the first stage (they are determined from the graph for i = 0). The velocity scale is $V_x = \sqrt{2E_0/m}$. The computations were performed for $x_* = 0.2$ m, $L_0 = 4 \cdot 10^{-8}$ H, $E_0 = 5 \cdot 10^5$ J, $m = 10^{-2}$ kg, $k = 4 \cdot 10^{-7}$ H/m, $n = 2$, and $\lambda = 1$.

3. The whole preceding investigation of the problem of a rail accelerator was without taking account of any constraints associated with the real characteristics of materials of elements of the accelerator and the body being accelerated. It is well known that the possibility of transmitting large currents in an acceleration channel is constrained by requirements on the mechanical and thermal strength of the electrode material. These problems

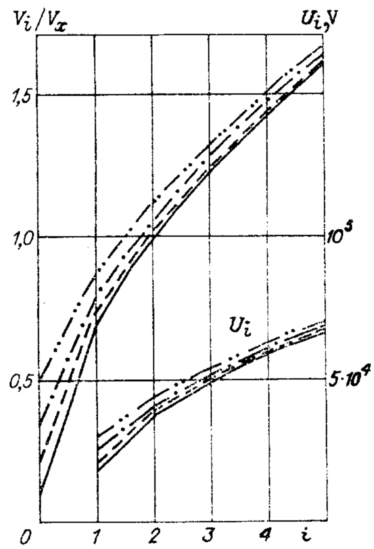

Fig. 1

occur even for the material of the body being accelerated. Let us consider the corrections that must be introduced into the system of estimates presented above in connection with taking account of these constraints.

An estimate of the magnitude of the mean acceleration (for $V_0 = 0$) yields

$$a = V_*^2/2x_*, \tag{3.1}$$

and the condition assuring the strength of the cylindrical body being accelerated is formulated as

$$V_* \rho \Delta / 2x_* < \sigma_T, \tag{3.2}$$

where σ_T, ρ are the yield point and density of the material of the body being accelerated, and Δ is its dimension in the acceleration direction. Taking account of relationships of the type (1.11), we have a condition on the maximally allowable energy reserve in the condenser

$$E_0 < 2\sigma_T S x_* = 2A_* \tag{3.3}$$

(S is the transverse section of the body being accelerated and A_* is the work of the force destroying the body on the acceleration path).

Therefore, taking account of the possible destruction of a body being accelerated to a velocity V_* (in the $n = 2$ mode) in the section x_* explicitly constrained the allowable storage energy reserve by a quantity proportional to the work of the "destructive" force on the acceleration path. Numerical data characterizing the orders of the boundary energies can be presented. Thus $E_0 = 3 \cdot 10^4$ J to accelerate a titanium body [6] for $x_* = 0.2$ m, $S = 10^{-3}$ m^2 (a disk with about a 2-cm radius). To accelerate a metal–ceramic body in this case E_0 should be almost two orders of magnitude greater [6].

Calculations analogous to those performed result for a multistage accelerator in the condition

$$\nu_i^2 - 1 < 2\sigma_T x_* / (V_{i-1}^2 \rho \Delta). \tag{3.4}$$

The constraint on the electrode strength, written in [4] in the form $\sigma_e > kI_0^2/2S_e$, with a certain strength margin can be represented as follows (I_0 is the storage short-circuit amplitude current)

$$\sigma_e > kU_0 C^2 \omega_{00}^2 / 2S_e \tag{3.5}$$

(σ_e is the yield point of the electrode material). Therefore, two more have been added to the relationships obtained earlier. If it is taken into account that the problem of an ideal rail accelerator includes more than ten characteristics, then it can be shown that by having just four relationships it is difficult to look forward to obtaining estimates of the parameters assuring an optimal mode of accelerator operation. However, if it is taken into account that part of the characteristics is given very specifically (the velocity, length, n, etc.), the number of free characteristics can be diminished and the express-estimate described can aid in rapidly and sufficiently exactly selecting those modifications that would be considered close to optimal by some criteria. Their further (more exact) numerical investigation permits selection of the necessary modification.

4. In conclusion, let us note that this approach makes possible the estimation of the influence of the active resistance of the accelerator channel. In this case (1.3) takes the form

$$\ddot{q} + \left[\frac{2n + (2\tau + \gamma n)\, t_*}{(n+2)\, t_*}\right]\dot{q} + \frac{2q}{L_0 C\,(n+2)} = 0, \tag{4.1}$$

where γ is the ratio between the linear resistance and the linear inductance of the rails accelerator channel, τ is the ratio between the initial value of the loop resistance and the initial value of the inductance. The representation (4.1) permits making computations similar to those performed in the previous sections. It is hardly expedient to present them here since for the t_* for which the discharge is still quasisinusoidal, the results

TABLE 2

$2E_0/mV_x^2$	η_{est}	η true
$5{,}55\cdot 10^{-2}$	0,05	0,08
0,111	0,09	0,13
0,287	0,185	0,196
0,555	0.28	0.24

obtained from the efficiency and the requirement on the storage loop eigenfrequency will be corrected only quantitatively. A study of the aperiodic mode is of definite interest.

The simplest analysis of (4.1) shows that each acceleration mode is made aperioidic sooner or later. A condition can be formulated for which the damping coefficient effective during acceleration will not permit development of a vibrational process generally. It is easy to see that this condition is the following

$$\sqrt{\frac{2}{(n+2)}}\,\omega_{00} \approx \frac{2n + (2\tau + \gamma n)\,t_*}{2\,(n+2)\,t_*}. \qquad (4.2)$$

After a time t = 6/δ in the discharge aperiodic mode the current drops e times as compared with the amplitude value. Let us limit the acceleration duration to precisely this quantity. Then

$$t = 4(n+3)/(2\tau + \gamma n) \qquad (4.3)$$

and we have when using a constant effective value of the acceleration as before

$$V_* = 2x_*/t_* = nL_0\,(2\tau + \gamma n)/(2k\,(n+3)). \qquad (4.4)$$

Introducing the characteristic velocity as in [5]

$$V_x = 4\langle R\rangle/k = 2L_0(2\tau + \gamma n)/k \qquad (4.5)$$

and using the relationship presented there, we obtain that the ratio between the kinetic and the scattered energies due to Joulean heating is expressed as

$$\mu \equiv E_k/E_R = V_*/V_x = n/(4\,(n+3)). \qquad (4.6)$$

In other words, a purely aperiodic current discharge mode during acceleration permits utilization of just about 1/5 of all the storage energy in the best case. For $n = 2$, $\mu = 0.1$.

Let us estimate the magnitude of the efficiency of a non-ideal rail accelerator as a function of the storage energy and the accelerator characteristics cited in [4, 5]. For a typical rail accelerator we have $L_0 = 10^{-8}$ H, $k = 10^{-7}$ H/m, and $\tau = 10^5$ sec^{-1} (the linear resistance of the acceleration channel is here taken equal to 10^{-2} Ω/m, and the storage resistance is 10^{-3} Ω). The nature of the change in the efficiency of such an accelerator is represented in Table 2 as a function of $2E_0/mV_x^2$. The calculations were performed for n = 1 with (4.5) taken into account. Here the values are presented of the efficiencies obtained for a numerical solution of the rail accelerator differential equations with the characteristics presented above. The numerical solution also showed that at the time when n = 1 the acceleration in a real rail accelerator practically ceases, which also justifies the selection of the value of n.

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SINGULARITIES OF THE ROTATING CYLINDRICAL SHELL CONVERGENCE PROCESS

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It is noted in [1, 2] that during the inertial convergence of viscous cylindrical shells the inner shell boundary is arrested upon reaching a certain distance from the axis of symmetry r_{min} whose magnitude depends on the coefficient of viscosity and on the geometric and kinematic shell parameters. This dependence can be utilized to determine the coefficient of viscosity. However, measurements are made difficult because of the small r_{min} for a sufficiently high convergence rate.

Inertial convergence of a rotating cylindrical shell is investigated in this paper with compressibility and viscosity taken into account. Energy transformation and redistribution occurs during convergence of such a shell. The kinetic energy of radial motion is converted into rotation energy and into internal energy of the substance. The energy of the rotational motion goes over into thermal energy due to the viscous friction of the rotating shell layers. A time sets in here when the velocity of radial motion of the inner shell boundary becomes zero, the inner boundary is arrested at a certain distance from the axis of symmetry, after which separation of the shell starts.

The quantity r_{min} depends on the shell geometric dimensions, on the relationship of the kinetic energy of the radial and rotational motion at the initial time, and what is of special interest, on the coefficient of viscosity of the shell material. Depending on the initial data, r_{min} for a viscous rotating shell can turn out to be substantially greater than for a shell of the same dimensions that does not rotate.

The system of equations describing rotating shell motion with viscosity and compressibility taken into account has the following form [3]

$$\begin{gathered}\rho\, du/dt = -\partial P/\partial r + \partial S_{rr}/\partial r + (S_{rr} - S_{\varphi\varphi})/r + \rho\omega^2 r,\\ d\omega/dt = -2u\omega/r + (\partial S_{r\varphi}/\partial r + 2S_{r\varphi}/r)/r,\\ d\rho/dt = -\rho(\partial u/\partial r + u/r),\ dr/dt = u,\\ de/dt = -P\, d1/\rho/dt + (S_{rr}^2 + S_{\varphi\varphi}^2 + 2S_{r\varphi}^2)/2\rho\mu,\\ P = P(\rho, e),\ S_{rr} = \mu(2\partial u/\partial r - u/r)2/3,\\ S_{\varphi\varphi} = (2/3)(\mu(2u/r - \partial u/\partial r)),\ S_{r\varphi} = \mu r\, \partial\omega/\partial r,\end{gathered} \tag{1}$$

where ω is the angular rotation, μ is the coefficient of viscosity, S_{ij} are the viscous stress tensor deviator components in an x, r, φ coordinate system (ox is the axis of rotation), and the remaining notation is standard.

Heat conductivity and second viscosity effects are not taken into account in this formulation. For the case of condensed media under consideration such an approximation is justified. The heat conductivity effects are excluded since the thermal relaxation time $\tau = \ell^2/\chi$ is substantially greater than the characteristic times of the process [3, 4] (ℓ is the characteristic dimension and χ is the thermal diffusivity coefficient). The contribution of second viscosity to the global part of the stress tensor is small compared with the pressure [5].

The system (1) was solved numerically by a finite-difference method using the method of splitting according to physical processes [6, 7]. The solution of the system (1) was

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